MEASUREMENT OF HALF-LIFE

The radioactivity of a short-lived radionuclide is measured as a function of time and the half-life of the substance is determined.

Theory:

The Geiger-Muller Counter

The counter consists of a cylindrical chamber (tube) with a wire stretched along its longitudinal axis. The chamber walls act as the cathode, and a positive voltage is applied to the wire (usually tungsten), which is insulated from the cylinder walls, making it the anode. The cylinder is filled with a low pressure gas mixture of argon and ethyl alcohol.

When an ionizing particle passes through the gas in the counter it liberates electrons by collision, and these are attracted toward the center wire (anode). The electrons striking the anode and the change in tube voltage due to the ion sheath resulting from electron-gas atom collisions produce a pulse at the anode which is amplified and counted.

The counter is inoperative until the ion sheath has been dissipated. The time required for this dissipation to occur is called the DEAD TIME of the counter and is typically of the order of hundreds of microseconds.

A number of factors must be taken into account when analyzing counting experiments. Three of these factors are background, counter losses, and statistics.

Background is the term applied to counts that are registered even when no radiation source is present. This is due to contamination in the lab from other experiments, building materials, the soil, and cosmic rays. The background counting rate must be subtracted from the total rate to obtain that due to the radiation source alone.

As already mentioned, during the development of a discharge, the voltage at the anode is lowered to such an extent by the ion sheath that a second particle passing through at that time will not be counted. This "dead" time is denoted τ_D. If m' is the observed counting rate, the actual rate is given approximately by

$$m = \frac{m'}{1 - m' \tau_D}$$

Since τ_D is small, of the order of 100 microseconds, this correction usually need only be applied for counting rates higher than 100/sec (i.e. when $m' \tau_D$ is significant compared to 1, say $m' \tau_D > 0.01$).

The count rate obtained in any time interval will fluctuate from the average counting rate over a long period of time according to the laws of probability. Counting experiments involving radioactive decay or gamma radiation absorption obey a Poisson statistical distribution. For a single measurement of n counts the standard deviation is equal to $\sqrt{n}$, and the result of a measurement is quoted as $n \pm \sqrt{n}$. To obtain a relative uncertainty of 1%, for example, one must have $\sqrt{n}/n = 0.01$, or $n = 10{,}000$ counts. One must always count long enough to obtain sufficient counts in order to have the desired accuracy. If N runs are made under the same conditions, the results are given by $\bar{n} \pm \sqrt{\bar{n}/N}$. That is, the standard deviation of the mean of N runs is reduced by a factor of $\sqrt{N}$. The probable error for a single measurement is $0.67\sqrt{\bar{n}}$. In other words, if the deviation from the mean $(n - \bar{n})$ is calculated for a series of measurements made under identical conditions, half of the deviations should be greater than $0.67\sqrt{\bar{n}}$ and half should be less.

Radioactive Decay (Determination of Half-life)

Radioactive decay, like gamma ray absorption, is a statistical phenomenon: every nucleus in a sample of radioactive material has a certain probability of decaying, but there is no way of knowing in advance which nuclei will actually decay in a particular time span.

Let λ be the constant probability per unit time for the decay of each nucleus of a given substance. Since λ is the probability per unit time, $\lambda\Delta t$ is the probability that any nucleus will decay in a time interval Δt. If a sample contains $N(t)$ undecayed nuclei, the number ΔN that decay in a time Δt is the product of the number of nuclei $N(t)$ and the probability $\lambda\Delta t$ that each will decay in Δt.

i.e.

$$\Delta N = -N(t)\,\lambda\,\Delta t \tag{2}$$

The minus sign is required since $N(t)$ is decreasing with time (i.e. ΔN is intrinsically negative). Note that because $N(t)$ continuously decreases with time, equation (2) cannot be solved because the variation of N with t is unknown (in fact, determining $N(t)$ is the purpose of this discussion). This problem can be overcome by taking the infinitesimal limit of the time interval, i.e. $\Delta t \to dt$. Then $\Delta N \to dN$ and $N(t)$ can be replaced by N, the number of nuclei at the beginning of the infinitesimal time interval, since dN is infinitesimally small during the interval dt.

$$dN = -N\,\lambda\,dt \tag{3}$$

and the number of undecayed nuclei remaining as a function of time can be found by integration.

$$\frac{dN}{N} = -\lambda dt \tag{4}$$

$$\int_{N_o}^{N} \frac{dN}{N} = -\lambda \int_{0}^{t} dt \tag{5}$$

$$\ln N = \ln N_o - \lambda t \tag{6}$$

$$N = N_o e^{-\lambda t} \tag{7}$$

The activity of a radioactive sample is given by

$$R = -\frac{dN}{dt} \tag{8}$$

By defining $R_o = \lambda N_o$,

$$R = R_o e^{-\lambda t} \tag{9}$$

That is, the count rate of radiation emitted from a radioactive source decays exponentially with time.

The half-life is the time interval over which the activity of the source falls to one-half its value at the beginning of the time interval. Let $t_{½}$ be the half-life. Then

$$½R_i = R_i e^{-\lambda t_{1/2}} \tag{10}$$

where R_i is the activity at the beginning of the time interval, and

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \tag{11}$$

Apparatus:

The apparatus consists of a PASCO SN-7927A Geiger-Muller counter tube/Power Supply, an Xplorer GLX interface, and a computer.

The GLX supplies voltage to the G-M tube, counts the pulses from the anode, and provides the interface between the G-M tube and the PC. The high voltage for the G-M tube is pre-set by its integrated power supply.

The radiation source used in this experiment is a liquid ^{137}Ba source produced by elution in a ‘generator’. Care should be taken not to spill the ^{137}Ba solution.

^{137}Ba Source Generator

The radioisotope generator is a commercial unit. It contains a long-lived 'parent' isotope, ^{137}Cs (half-life of 30 years) and short-lived 'daughter' isotope, ^{137}Ba*, whose half-life is to be measured, which are absorbed into a small ion exchange resin column. Both the ^{137}Cs and the resin are insoluble in the eluent, a solution of hydrochloric acid and sodium chloride. However, the daughter radionuclide ^{137}Ba* resulting from the beta decay of ^{137}Cs is soluble. Thus when the eluent is passed through the column of the generator the liquid contains only the daughter radionuclide. ^{137}Ba* decays to its ground state by the emission of a 0.662 MeV gamma ray which is detected and counted by the Geiger-Muller tube/GLX. The radioactive decay scheme for ^{137}Cs is shown in Figure 1.

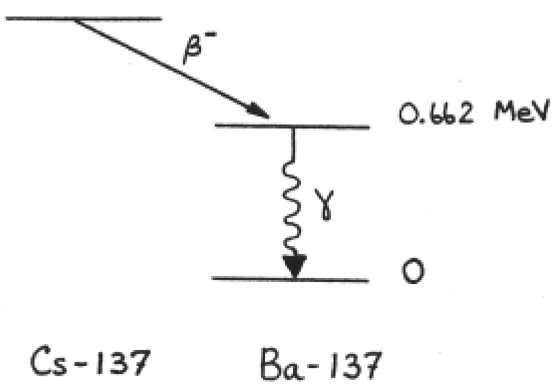

Figure 1

Procedure and Experiment:

**** DO NOT TOUCH THE END OF THE G-M TUBE – it is extremely fragile. ****

1. Turn on the computer and click the ‘modphys’ user name. If necessary, wait for Sophos to finish updating. (Updating is complete when the shield in the system tray is solid blue.
2. Turn on the Xplorer GLX (green button in bottom right corner).
3. The DataStudio program should load automatically once the Xplorer GLX is on.
4. In the ‘Choose sensor or instrument…’ window select ‘General Counting’ and click OK.
5. Before doing the half-life measurement, the background radiation must be measured. Be sure that all radiation sources are removed from the vicinity of the G-M tube.
6. Background radiation is counted as follows:
   a) Click the Setup button in the toolbar, the Experiment Setup window will open.
   b) Click the ‘Constants’ tab and set the count interval to 300 seconds.
   c) Close the Experiment Setup window.
   d) Click ‘Start’.
   e) A table window will open automatically and show the data collected during each count interval.
   f) Click ‘Stop’ after one count interval (300 seconds) is complete and record the number of counts detected.
7. To prepare to collect the half-life data, click Setup and set the count interval time to 30 seconds.
8. The ^{137}Ba sample is prepared as follows (ask the instructor to do this):
   a) Obtain the generator kit and the bottle of eluting solution.
   b) Place a metal disk planchet in the larger, rectangular metal tray.

   c) Attach the plastic tube to the syringe and draw eluting solution into the syringe.
   d) Remove the tube from the syringe and remove the plastic stoppers (top and bottom) from the generator.
   e) Insert the syringe firmly into the hole on the top of the generator. While holding the generator over the planchet, slowly and steadily push on the syringe plunger to force solution through the generator until the planchet is full.
   f) Slide the planchet and tray under the G-M tube and **immediately** click ‘Start’.
   g) In addition to viewing the data in the table window, a real-time graph of the data can be viewed by double-clicking ‘Graph’ in the Display window (bottom left of screen).
   h) Click ‘Stop’ after an elapsed time of 12 minutes.

9. Either record your count and data manually in your notebook or export the data electronically.
10. To export the data electronically:
    a) Go to the File menu and select ‘Export Data…’.
    b) Give the data file a descriptive name (e.g. halflife_data_20071024.txt).
    c) Open the tab-delimited text data file in Excel, save it as an Excel file, and email it to all the members of the lab group.
11. Once the experiment has been completed, turn off the computer (the Xplorer GLX will turn off automatically once its battery is fully re-charged).

Analysis:

When tabulating the data, convert the count rate (counts in 30 sec) to counts/sec. It may be necessary to correct for dead time at the high initial count rates. Use a value of 100 microseconds for the dead time for the G-M tube. Remember to also correct for background. Tabulate the corrected count rates (count/sec).

Plot the natural logarithm of the corrected count rates versus elapsed time.

Is the theoretical relation between count rate and elapsed time verified?

Determine the half-life of ^{137}Ba from your graph. Compare to the accepted value of 2.55 min (153 sec).

References:

Bleuler & Goldsmith, Experimental Nucleonics, QC 784
Fretter, Introduction to Experimental Physics, QC 41
Halliday, Introductory Nuclear Physics, QC 173
Melissinos, Experiments in Modern Physics, QC 33
Taylor, An Introduction to Error Analysis, QA 275