SUMMARY OF PRINCIPLES

LINEAR MOTION KINEMATICS (constant acceleration)

$$x = v_0 t + \tfrac{1}{2}at^2$$
$$v = v_0 + at$$
$$v^2 = v_0^2 + 2ax$$

To apply in two dimensions, choose an *x*-*y* coordinate system so that the acceleration coincides with either the *x* or *y* direction. Then the motion along the *other* coordinate direction is at constant velocity (component of acceleration in that direction is 0). The components of motion in the *x* and *y* directions are analyzed separately. The components of motion can be combined to give magnitude and direction by using Pythagoras and the arctangent function.

NEWTON'S FIRST LAW (EQUILIBRIUM):

If an object is at rest or moving at constant velocity (constant speed in a straight line), there is no NET force acting on it. i.e. The vector sum of all the forces acting on the object is 0.

$$\sum \vec{F} = 0 \text{ which means } \sum F_x = 0 \text{ and } \sum F_y = 0$$

NEWTON'S SECOND LAW (DYNAMICS)

If there <u>is</u> a net force acting on an object, then it will be accelerating. The acceleration is given by:

$$\vec{a} = \frac{\sum \vec{F}}{m} \quad \text{so} \quad \sum \vec{F} = m\vec{a} \quad \text{and} \quad \vec{a}, \vec{F} \text{ have the same direction}$$

Choosing an *x*-*y* coordinate system so that the acceleration is in the *x*-direction, for instance, yields:

$$\Sigma F_x = ma \quad \text{and} \quad \Sigma F_y = 0$$

UNIFORM CIRCULAR MOTION

For an object moving in a circular path at constant speed v there is a centripetal acceleration, a_c , directed toward the centre of the circle:

$$a_c = \frac{v^2}{r}$$

and this acceleration <u>must</u> be produced by a net centripetal force directed radially inward such that

$$F_c = \sum F_{\text{radial}} = m\frac{v^2}{r}$$

CONSERVATION OF <u>MECHANICAL</u> ENERGY

The change in the kinetic energy of a system equals the <u>net</u> work done on the system (Work-Energy Theorem). The net work is the algebraic sum of the work done due to all the forces acting on the system, or equivalently, the work done by the net force acting on the system.

$$\Delta KE = \Delta W_{net}$$

The work done by a force is given by $W = (F\cos\theta)\, s$ where θ is the angle between the force and the displacement of the object.

If the net work done on a system is due only to forces for which a potential energy can be defined (e.g. conservative forces such as gravity, the ideal spring force, the electric force), then <u>Mechanical Energy is Conserved</u>.

$$E_o = E_f$$
$$\text{KE}_o + \text{PE}_o = \text{KE}_f + \text{PE}_f$$

In general, to include non-conservative forces:

$$W_{nc} = \text{KE}_f + \text{PE}_f - (\text{KE}_o + \text{PE}_o) = E_f - E_o$$

where W_{nc} = work done by the non-conservative forces (e.g. friction)
In the case of friction, W_{nc} is negative because the force of friction is directed opposite to the displacement (angle between frictional force and displacement is 180°)

CONSERVATION OF MOMENTUM

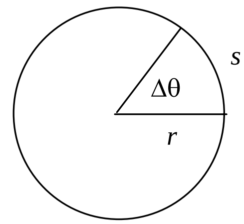
If a system is <u>not</u> acted on by a net <u>external</u> force, then the total momentum of the system is <u>constant</u>. (The momentum of an object with mass m moving with velocity $\vec{v}$ is $\vec{p} = m\vec{v}$.)

$$\vec{P}_f = \vec{P}_o$$
$$\text{i.e. } p_{f1x} + p_{f2x} + p_{f3x} + \ldots = p_{o1x} + p_{o2x} + p_{o3x} + \ldots$$
$$\text{and } p_{f1y} + p_{f2y} + p_{f3y} + \ldots = p_{o1y} + p_{o2y} + p_{o3y} + \ldots$$

i.e. the total momentum is conserved in <u>each</u> component direction.

ROTATIONAL KINEMATICS

Consider a point on the rim of a wheel of radius *r*, or an object moving in a circular path of radius *r*. Recall $s = r\Delta\theta$ (definition of radian angle measure). If the wheel is turning, or the object moving (in the circular path), then $v = s/\Delta t$ where *s* is the distance travelled along the circle in time Δt.



At any particular instant the velocity is tangent to the circular path.

$$v = \frac{s}{\Delta t} = \frac{r\Delta\theta}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

$\Delta\theta/\Delta t$ is the rate of rotation. A valid unit for $\Delta\theta/\Delta t$ is revolutions per minute, although the SI unit is radians/sec. $\Delta\theta/\Delta t$ is given the symbol ω and is called the angular velocity.

$$v = r\omega$$

If the wheel (or object) is spinning faster and faster or slower and slower, then a point on the rim is accelerating. The (tangential) acceleration is given by

$$a_t = \frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = r\alpha$$

where α is called the angular acceleration. We can write the kinematic equations for a point on the rim as:

$$s = v_0 t + ½ a_t t^2$$

$$v = v_0 + a_t t$$

$$v^2 = v_0^2 + 2a_t s$$

However, points at other parts of the wheel will have different *s*, *v*, and a_t depending on their distances from the centre of the wheel. If we recall that $s = r\theta$, $v_t = r\omega$, and $a_t = r\alpha$, then we can write the kinematic equations using the angular variables:

$$\theta = \omega_0 t + ½\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

and these equations apply to all points on the wheel, not just at the rim.

ROTATIONAL DYNAMICS

If an object is undergoing angular acceleration there must be a net torque acting on it, such that

$$\Sigma\tau_{ext} = I\alpha \quad \text{where } I = \text{moment of inertia} = \Sigma mr^2$$

Torque τ is defined as:

$\tau = F\ell$ where the moment arm ℓ is the perpendicular distance between the line of action of the force **F** and the axis of rotation.

If an object is in equilibrium, then $\Sigma\mathbf{F}_{ext} = 0$ and $\Sigma\tau = 0$

ROTATIONAL WORK AND KINETIC ENERGY

The rotational work done by a constant torque τ acting through an angular displacement θ is $W_R = \tau\theta$.

From the work-energy theorem, $KE_R = \frac{1}{2}I\omega^2$ is the kinetic energy of an object with angular speed ω and moment of inertia *I*.

The kinetic energy of a rigid, extended body is the sum of its translational kinetic energy and its rotational kinetic energy. If an object is rolling without slipping, the translational speed and angular velocity are related by

$$v = r\omega.$$

ANGULAR MOMENTUM

$$L = I\omega$$

where *L* = angular momentum of object with moment of inertia *I* rotating about a fixed axis with angular velocity ω.

The total angular momentum of a system remains constant if the net external torque acting on the system is 0:

$$I_f\omega_f = I_o\omega_o \text{ when } \Sigma\tau_{ext} = 0$$

SIMPLE HARMONIC MOTION

– occurs due to a Hooke's Law Force (a restoring force),

F = – constant × displacement from equilibrium

$F = -kx = ma$ (from Newton's 2nd Law)

Kinematic Equations for SHM:

$$x = A\cos(\omega t)$$

$$v = -A\omega\sin(\omega t)$$

$$a = -A\omega^2\cos(\omega t)$$

where A = maximum displacement = amplitude

$$v_{max} = A\omega = A\left(\frac{2\pi}{T}\right) = A\sqrt{\frac{k}{m}} = \text{ maximum speed}$$

$$a_{max} = \frac{k}{m}A = \left(\frac{4\pi^2}{T^2}\right)A = A\omega^2 = \text{ maximum acceleration}$$

Note that *x* and *a* have maximum magnitude when displacement is at maximum and speed is 0. *v* has maximum magnitude when object is passing through equilibrium position (where displacement and acceleration are 0).

Examples of SHM:	Mass on ideal spring	Simple Pendulum
Angular Frequency	$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{L}}$
Oscillation Period	$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$
Oscillation Frequency	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

ENERGY RELATIONS IN SHM

$$E = \text{KE} + \text{PE} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\text{max}}^2$$

KE = maximum when PE = 0 (at $x = 0$, equilibrium position)
PE = maximum when KE = 0 (at $x = A$, maximum displacement)

PRESSURE AND DEPTH IN A STATIC INCOMPRESSIBLE FLUID

$$P_2 = P_1 + \rho g h$$

where P_2 is the pressure at a depth h below a point where the pressure is P_1.

BUOYANT FORCE:
An object wholly or partially submerged in a fluid feels an upward buoyant force equal to the weight of the volume of fluid that has been displaced:

$$F_B = W_{\text{fluid}} = \rho_L g V_{\text{fluid}}$$

where ρ_L is the fluid density and V_{fluid} is the volume of fluid displaced.

If the object is completely submerged: $V_{\text{fluid}} = V_{\text{object}}$
If the object is floating: F_B = weight of object = $M_{\text{object}}g = \rho_{\text{obj}}V_{\text{obj}}g$

INCOMPRESSIBLE FLUID FLOW:

Q = volume flow rate = constant
$A_1v_1 = A_2v_2$

BERNOULLI'S EQUATION:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

POISEUILLE'S LAW (VISCOUS FLOW)

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta L}$$

HARMONIC WAVES (SINE WAVE)
– produced by a SHM oscillator
Wave displacement as a function of position and time is given by:

$$y(x,t) = A\sin\left(2\pi f t \mp \frac{2\pi x}{\lambda}\right)$$

v = wave speed = $\lambda/T = f\lambda$
T = period = time for one complete oscillation
λ = wavelength = distance between points on wave that have identical characteristics. e.g. distance between consecutive crests

If a wave is propagating uniformly from a point source, the intensity, I, (= power flowing through unit area) is

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

where r = distance from the source
and $A = 4\pi r^2$ = area of the sphere through which the power is passing.
If energy losses are negligible, then P is a constant (energy is conserved) and

$$I \propto \frac{1}{r^2} \quad ; \quad \frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

DOPPLER EFFECT
– change in frequency associated with the relative motion of a wave source and observer

$$f_o = f_s \frac{\left(1 \pm \frac{v_o}{v}\right)}{\left(1 \mp \frac{v_s}{v}\right)}$$

where
f_o = observed frequency
f_s = source frequency
v_o = observer speed
v_s = source speed
v = wave speed

The upper sign is used when the observer moves toward the source or the source moves toward the observer, and the lower sign is used for motion away.

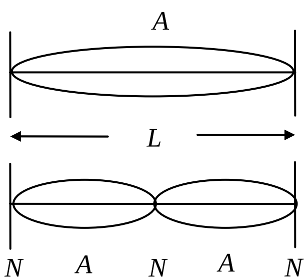
CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE
For two wave sources vibrating in phase, a difference in path lengths that is an integer number of wavelengths (λ, 2λ, 3λ, ...) leads to constructive

interference; a difference in path lengths that is an odd multiple of half the wavelength ($\lambda/2$, $3\lambda/2$, $5\lambda/2$, ...) leads to destructive interference.

STANDING WAVES AND RESONANCE

When waves can only exist along a certain length L (e.g. guitar string, organ pipe) then only certain frequencies of waves can exist when the medium is made to vibrate by a disturbance or an oscillator. The resonant frequencies are determined by the boundary conditions, i.e. by what happens at the ends of the medium.

For a string fixed at both ends, the ends must be nodes.



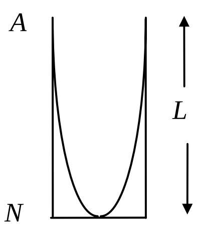
$$L = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L \Rightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

$$L = 2\left(\frac{\lambda_2}{2}\right) \Rightarrow f_2 = 2\left(\frac{v}{2L}\right)$$

Possible resonant frequencies are: $f_n = n\left(\frac{v}{2L}\right)$; $n = 1, 2, 3, \ldots$

Sound Waves in Pipes:

CLOSED PIPE (closed at one end, open at the other)

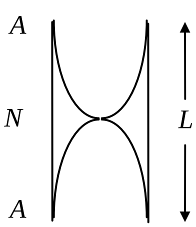


L = odd multiple of $\lambda_n/4$

$$L = \left(\frac{2n+1}{2}\right)\frac{\lambda}{4} \; ; \; n = 0, 1, 2, \ldots$$

$$f_n = \left(\frac{2n+1}{2}\right)\frac{v}{4L}$$

OPEN PIPE (open at both ends)



$$L = n\left(\frac{\lambda_n}{2}\right)$$

$$f_n = n\left(\frac{v}{2L}\right)$$

ELECTROSTATICS

Effects due to charge distributions are determined by calculating the effect due to each charge, and then adding these individual effects (vector sum for force and electric field, algebraic sum for potential).

Electrostatic force between charges q_1 and q_2 separated by distance r is:

$$F = k\frac{|q_1 q_2|}{r^2} \qquad k = 9.00 \times 10^9 \text{ N·m}^2/\text{C}^2$$

The force is directed along the line between the two charges; and is repulsive if the charges are both positive or both negative, and attractive if one charge is positive and the other negative.

ELECTRIC FIELD:

The electric field at a point in space is the force that would act on a unit positive charge placed at this location. (i.e. $\vec{E}$ = force per unit charge)

The force acting on a charge q_o placed where there is an electric field $\vec{E}$ is

$$\vec{F} = q_o \vec{E}$$

Note that the force is in the same direction as the electric field if q_o is positive, and is in the opposite direction to the electric field if q_o is negative.

The magnitude of the electric field **produced by** a point charge q is

$$E = \frac{kq}{r^2} \quad \text{where } r \text{ is the distance from } q$$

$\vec{E}$ points away from a positive charge and toward a negative charge.

ELECTRIC POTENTIAL:

The electric potential at a point in space is the potential energy that a unit positive charge would have if placed at this location. (V = EPE/q_o = EPE per unit charge)

A charge q_o placed where there is a potential of V has an electrostatic potential energy:

$$\text{EPE} = q_o V$$

If a charge q_o moves through a potential difference of ΔV volts, then the change in the electrostatic potential energy of this charge is

$$\Delta\text{PE} = q_o \Delta V$$

From energy conservation:

$$E_f = E_o$$

$KE_f + EPE_f = KE_o + EPE_o$
$EPE_o - EPE_f = KE_f - KE_o$
$-(EPE_f - EPE_o) = KE_f - KE_o$
$-\Delta EPE = \Delta KE$
and $\Delta EPE = q_o \Delta V$

i.e. a charge q_o moving through a potential difference loses EPE and gains an equivalent amount of KE.

Unit of electronvolt (eV):
When the charge on an object is a small multiple of the elementary charge, *e*, it is often convenient to express the charge in units of *e* rather than Coulombs, and energy in units of electronvolts rather than Joules.

e.g. The change in potential energy when an object with a charge of +4*e* is moved through a potential difference of 5 Volts is:
$\Delta EPE = q\Delta V = (4e)(5\text{V}) = 20 \text{ eV}$

The 'eV', called the electronvolt, is a valid unit of energy. The conversion factor for eV to Joules is obtained by substituting the value for *e*:
$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(\text{V}) = (1.60 \times 10^{-19} \text{ C})(\text{J/C})$
$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$

ELECTRIC POTENTIAL **<u>DUE TO</u>** POINT CHARGES
Defining the electric potential to be 0 at an infinite distance from a point charge *q*, the electric potential at a distance of *r* from a point charge *q* is

$$V = \frac{kq}{r}$$

RELATION BETWEEN ELECTRIC FIELD AND CHANGE IN ELECTRIC POTENTIAL
Consider a point charge $+q_o$ placed in a **uniform** electric field. The charge is moved a distance Δs (from point A to point B) along an electric field line, in the direction of the field. The change in potential , ΔV (= $V_B - V_A$) equals the negative of the work done by the electric force acting on the charge divided by the charge (negative of work done per unit charge).

$$\Delta V = -\frac{W_{el}}{q_o} = -\frac{F_{el}\Delta s \cos 0^o}{q_o} = -\frac{q_o E \Delta s}{q_o} = -E\Delta s$$

$$E = -\frac{\Delta V}{\Delta s}$$

ELECTRIC CIRCUITS
Voltage, Current, and Resistance for a circuit or component are related by

$$R = \frac{V}{I} \quad ; \quad V = IR \quad ; \quad I = \frac{V}{R}$$

If the resistance, *R*, of a component is constant over a range of voltage and current values, the above relations are called Ohm's Law.

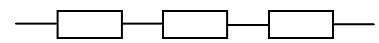
The power dissipated in a component carrying current *I*, across which there is a voltage drop *V* is:

$$P = VI$$

If the component is a resistor, then

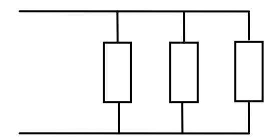
$$P = VI = I^2 R = \frac{V^2}{R}$$

COMPONENTS IN SERIES:



- have same current
- total voltage drop = sum of individual voltage drops
- have equivalent resistance of R_{ser} where $R_{ser} = R_1 + R_2 + R_3 + ...$

COMPONENTS IN PARALLEL:



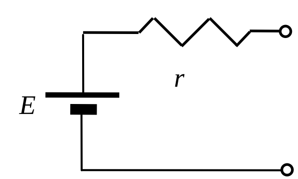
- have same voltage drop
- total current = sum of individual currents
- have equivalent resistance of R_{par} where

$$\frac{1}{R_{par}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

KIRCHHOFF'S LAWS:
Current Law
Total current entering a point in a circuit = total current leaving this point

Voltage Law
In a complete loop in a circuit, the sum of the applied emf's equals the sum of the voltage drops.

REAL VOLTAGE SOURCE:
A real voltage source (as compared to an ideal one) has an internal resistance *r*. A real voltage source is represented as an ideal source in series with a resistance:



MAGNETISM
In general, moving charges, whether in the form of moving charged particles or current in a wire, feel a force when there is an external magnetic field. This force is called the Lorentz Force.

The force on a particle of charge q_o moving with speed v in a magnetic field B is:

$$F = q_o vB \sin\theta$$

where θ is the angle between $\vec{v}$ and $\vec{B}$.
$\vec{F}$ is $\perp$ to $\vec{v}$ and $\vec{B}$ and its direction is given by a right-hand rule:
Extend the right hand so that the fingers of the right hand are pointing in the direction of the magnetic field and the thumb of the right hand is pointing in the direction of the velocity $\vec{v}$ of the charge. The palm of the hand is now pointing in the direction of the magnetic force that acts on a ***positive*** *charge. <u>If the charge is negative, the force is in the opposite direction</u>.*

When $\vec{B} \perp \vec{v}$, $\quad F = q_o vB$

Since $\vec{F} \perp \vec{v}$, when a charged particle moves into a region of uniform magnetic field then if $\vec{v} \perp \vec{B}$, the Lorentz force causes uniform circular motion.

From $F = ma$,

$$F_{\text{Lorentz}} = ma_c$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

The radius of the circular trajectory of the particle depends on its mass, speed, and charge, and on the magnetic field. The direction of curvature, or sense, of the trajectory depends on the sign of q and is given by the right-hand rule.

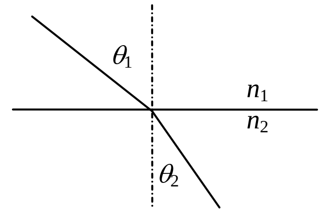
THE REFLECTION OF LIGHT
MIRRORS
Behavior is determined by Law of Reflection:

angle of incidence = angle of reflection

THE REFRACTION OF LIGHT
Light travels slower in a medium than in a vacuum.

$$n = \text{refractive index} = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

Snell's Law



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

If $n_2 > n_1$, then $\theta_2 < \theta_1$

Total Internal Reflection:
If $n_2 < n_1$, then for $\theta_1 = \theta_c$ = critical angle, $\theta_2 = 90°$ (no refraction occurs).

LENSES – behavior determined by Snell's Law of Refraction
Parallel light focusses (converging lens) or appears to originate (diverging lens) from a point a distance f (focal length) from the lens.

Principal Rays (lenses)
1. Ray parallel to principal axis refracts through, or seems to have come from, the focus.
2. Ray through or toward the focus refracts parallel to the principal axis.
3. Ray through the centre of the lens is undeviated.

Sign Convention (for single lens, object distance is positive):

	+	–
f	converging	diverging
d_o	object on same side of lens as incident light	object on opposite side of lens from incident light
d_i	image on opposite side of lens from incident light	image on same side of lens as incident light
m	upright image	inverted image

For multiple lens systems, image of 1st lens is object for 2nd lens, image of 2nd lens is object for 3rd lens, ...
Lens Equation:

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

$$\text{magnification, } m = \frac{\text{image height}}{\text{object height}} = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

Human Eye and Optical Instruments

The **normal near point** is the smallest object distance that the eye can accommodate. i.e. The smallest object distance at which the eye can maintain focus without strain. Often taken as 25 cm.

Optical instruments are used to obtain a larger retinal image than can be obtained by viewing the object directly with the unaided eye.

Angular Magnification

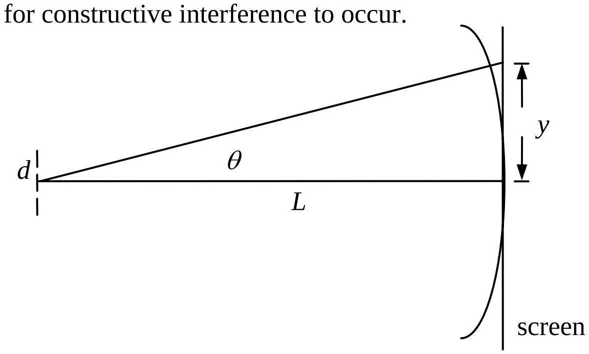
$$M = \frac{\text{retinal angle subtended by image from instrument}}{\text{angle subtended by object @ near point}}$$

PHYSICAL (WAVE) OPTICS

Interference of Light

Let λ = wavelength of light
d = slit separation
θ = angle of observation
L = screen distance
y = distance along screen from centre

Maxima, bright fringes, constructive interference occur when the difference in distances travelled by light from the different slits = $m\lambda$ (m = integer). i.e. The path length difference must be an integral number of wavelengths for constructive interference to occur.



The path length difference between consecutive slits spaced a distance d apart is $d \sin\theta$. $\therefore$ maxima occur when $d \sin\theta = m\lambda$

When θ is small, $\tan\theta$, which equals y/L, is approx. $\sin\theta$

$\therefore$ For small angles, maxima occur at screen positions y such that

$$d\left(\frac{y}{L}\right) = m\lambda$$

$$y = \frac{m\lambda L}{d}$$

MODERN PHYSICS

Thermal Radiation

The rate at which an object radiates thermal energy (e-m radiation) depends on its temperature (in Kelvin), its surface area, A, and its emissivity, e, by the Stefan-Boltzmann Law:

$$P_r = \sigma e A T^4$$

σ = Stefan's constant = 5.67×10^{-8} W/m^{2}K^4

An object also absorbs thermal radiation from its surroundings at a rate given by:

$$P_{abs} = \sigma e A T_{surr}^{\ 4}$$

Planck proposed the quantum theory in explaining the observations associated with blackbody radiation. Physical systems can only have certain energies, called quantum states. Planck's quantum theory assumed that the atomic oscillators emitting blackbody radiation could have only the discrete values of energy of $E = 0, hf, 2hf, 3hf, ..., nhf$

As applied to electromagnetic 'waves', the quantum theory becomes the photon (particle) theory of light.

The energy of a photon is given by:

$$E = hf = \frac{hc}{\lambda}$$

where h = Planck's constant = 6.63×10^{-34} J·s = 4.14×10^{-15} eV·s

The power (rate of energy) emission of a monochromatic light source can be expressed as:

P = energy emitted = # of photons emitted × energy per photon per unit time per unit time

Photoelectric Effect
Photons of light striking a material give their energy to electrons with which they ‘collide’. The maximum kinetic energy of the ejected electrons is given by the energy of the incident photon minus the work function of the material (the minimum energy required to free an electron).

$$KE_{max} = hf - W_o$$

Compton Scattering
Photons scattering from ‘free’ electrons lose energy in the collision. The loss of energy means an increase in wavelength. Applying conservation of energy and momentum to the photon-electron collision (a particle process):

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$

where $\Delta\lambda$ is the shift in the wavelength of the photons, m is the mass of the electron, and θ is the scattering angle.

Bohr Model of the Hydrogen Atom
Bohr applied the photon theory to the atom and also required that the angular momentum of the electron in its orbit around the proton was quantized (i.e. could only have certain values).

The quantization condition quantized the electron orbit radius and the energy of the atom:

$$r_n = \frac{n^2h^2}{4\pi^2mke^2} = (5.29\times10^{-11}\text{ m})n^2$$

$$E_n = -\frac{ke^2}{2r_n} = -\frac{2.18\times10^{-18}\text{ J}}{n^2} = -\frac{13.6\text{ eV}}{n^2}$$

The wavelength of the photon emitted or absorbed when the energy state of an atom changes from n_i to n_f is determined from

$$\Delta E = E_f - E_i = \frac{hc}{\lambda}$$

$$\frac{1}{\lambda} = \frac{13.6\text{ eV}}{hc}\left|\frac{1}{n_f^2} - \frac{1}{n_i^2}\right| = R\left|\frac{1}{n_f^2} - \frac{1}{n_i^2}\right|$$

R = Rydberg constant = $1.097 \times 10^7\text{ m}^{-1}$

X-ray Production
The maximum energy photons produced when electrons of $KE = eV_A$ collide with atoms and lose all their energy is:

$$E_{max} = KE = eV$$
$$hf_o = \frac{hc}{\lambda_o} = eV$$

NUCLEAR PHYSICS
The nucleus consists of neutrons and protons. The volume of the nucleus varies directly with the number of nucleons (neutrons and protons, A). Since for a sphere volume is $\propto R^3$, the radius of the nucleus varies as $A^{1/3}$.

i.e. $V \propto A$ so $R^3 \propto A$ so $R \propto A^{1/3}$

The nucleus is bound together by the strong nuclear force, which overcomes the electrostatic repulsion between the protons.

Since the nucleus is a bound system, energy must be added to a nucleus (i.e. work must be done on it) to separate it into its component neutrons and protons. This is called the BINDING ENERGY.

From $E = mc^2$, if the energy of the bound nucleus is less than that of the separate nucleons, then the ***mass*** of the bound nucleus must be less than the mass of its component nucleons. This is called the MASS DEFECT or MASS DEFICIT.

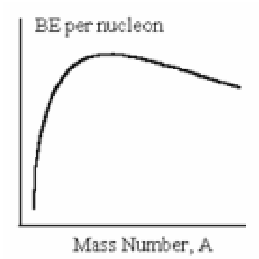
Since mass tables are in terms of the mass of the neutral atom (including Z electrons), rather than the bare nucleus, the binding energy, BE, is calculated as follows:

$$BE = (\text{Mass defect})c^2 = \left(Zm_H + Nm_n - {}^A_ZM\right)c^2$$

where ${}^{A}_{Z}M$ is the atomic mass and m_H is the mass of the hydrogen atom, (rather than the proton mass), to include the Z electrons.

Systematics of Stability

Additional neutrons keep the protons apart from each other, but as more neutrons are added they go into higher energy states.



Medium mass nuclei have the highest binding energy per nucleon (they are the most tightly bound).

Radioactivity

Unstable nuclei move toward stability by emitting particles and/or energy. Nucleon number, electric charge, energy, momentum, and angular momentum are all conserved in radioactive decay.

Alpha Decay

emission of an α particle (an α is a ^{4}He nucleus, 2 protons and 2 neutrons)

Beta Decay

The term beta decay is used to refer to two different processes: emission of an electron (β^-) and emission of a positron (β^+). In both cases, A does not change.

For β^- decay, a neutron changes into a proton and an electron, the electron being ejected from the nucleus. Z increases by 1 and N decreases by 1.

For β^+ decay, a proton changes into a neutron and a positron, the positron being ejected from the nucleus. Z decreases by 1 and N increases by 1.

Gamma Decay

release of e-m energy (as a photon) by an excited nucleus

Radioactive Decay

Radioactive decay is a random process. It is not possible to predict when a particular unstable nucleus will undergo decay.

If ΔN is the number of nuclei that decay in a short time interval Δt, then it is reasonable that ΔN should be proportional to the initial number of unstable nuclei, N, and the size of the time interval:

$$\Delta N \propto N\,\Delta t$$

Realizing that ΔN is negative (the number of radioactive nuclei remaining decreases with time), and defining λ as the (+) constant of proportionality:

$$\Delta N = -\lambda\, N\,\Delta t$$

The solution to this equation is:

$$N = N_o\, e^{-\lambda t}$$

where N_o is the number of unstable nuclei at $t = 0$, and λ is the **decay constant**.

Half-life

The half-life, $T_{1/2}$, of a radioactive material is defined as the time required for the number of radioactive nuclei to drop by a factor of 2. Half-life and decay constant are related by:

$$T_{1/2} = \frac{0.693}{\lambda}$$

Activity

The activity, A, of a radioactive sample is defined as the rate at which the radioactive nuclei are decaying.

$$A = -\frac{\Delta N}{\Delta t} = \lambda N = \lambda N_o\, e^{-\lambda t} = A_o\, e^{-\lambda t}$$

where A_o = initial activity = λN_o

Nuclear Fission

Since medium mass nuclei have higher BE/nucleon, if a heavy nucleus splits into medium nuclei, the higher BE/nucleon results in a release of energy.

Nuclear Fusion

Since medium mass nuclei have higher BE/nucleon, if two light nuclei can be made to join, the higher BE/nucleon results in a release of energy.