PART A

FOR EACH OF THE FOLLOWING QUESTIONS IN PART A, ENTER THE MOST APPROPRIATE RESPONSE ON THE OMR SHEET.

$a_c = \frac{v^2}{r}$; $v = \frac{2\pi r}{T}$

A1. The centripetal acceleration of an object undergoing uniform circular motion of radius r metres with a period of rotation of T seconds is given by:

$a_c = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r^2}{r T^2}$

(C) (A) $a_c = \frac{T}{2\pi r}$ (B) $a_c = \frac{T^2}{4\pi^2 r}$ (C) $a_c = \frac{4\pi^2 r}{T^2}$ (D) $a_c = \frac{T}{2\pi r^2}$ (E) $a_c = \frac{r}{T}$

$a_c = \frac{4\pi^2 r}{T^2}$

A2. To accelerate a car from rest to a speed v requires net work W_1. To accelerate the same car from speed v to speed $2v$ requires net work W_2. Which one of the following statements is correct? $W = \Delta KE$; $W_1 = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$; $W_2 = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2 = \frac{3}{2}mv^2$

(C) (A) $W_2 = W_1$ (B) $W_2 = 2W_1$ (C) $W_2 = 3W_1$ (D) $W_2 = 4W_1$ (E) $W_2 = ½W_1$

$\therefore W_2 = 3W_1$

A3. Which one of the following statements concerning non-conservative forces is <u>NOT</u> correct?

(E)
(A) Friction and air resistance are non-conservative forces. T
(B) An object can gain total mechanical energy because of work done by a non-conservative force. T
(C) The work done by a non-conservative force depends on the path taken by the object. T
(D) The work done by a non-conservative force is non-zero around a closed loop. T
(E) The work done by a non-conservative force is always negative. F

A4. It becomes necessary for you to jump out of a bedroom window. When you hit the ground you should flex your knees as you come to rest because $\vec{F}\Delta t = \Delta\vec{p}$ $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$; $\Delta\vec{p}$ is constant for this situation

(C)
(A) doing so will reduce your change of momentum.
(B) doing so will reduce the impulse that the ground exerts on you.
(C) doing so will <u>increase</u> the time over which your momentum changes, thereby <u>reducing</u> the average force exerted on you by the ground.
(D) doing so will <u>decrease</u> the time over which your momentum changes, thereby <u>reducing</u> the average force exerted on you by the ground.
(E) doing so will <u>increase</u> the time over which your momentum changes, thereby <u>increasing</u> the average force exerted on you by the ground.

A5. Two astronauts are floating in space, initially at rest. Astronaut 1, of mass m, pushes on astronaut 2, of mass $2m$. As a result of the push, astronaut 2 moves away from astronaut 1 with a speed v, and astronaut 1 Let the two astronauts be the system

(C)
(A) moves in the same direction as astronaut 2 with a speed v.
(B) moves in the opposite direction of astronaut 2 with a speed v.
(C) moves in the opposite direction of astronaut 2 with a speed $2v$.
(D) moves in the same direction as astronaut 2 with a speed $2v$.
(E) remains at rest.

$\sum \vec{F}_{ext} = 0$ so $\vec{P}_{tot}$ conserved

$\vec{P}_{tot_f} = \vec{P}_{tot_o}$

$m_1 v_{1f} + m_2 v_{2f} = 0$

$v_{1f} = -\frac{m_2 v_{2f}}{m_1} = -\frac{2m}{m} \cdot v = -2v$

continued on page 3...

A6. Two children are riding on a merry-go-round (a horizontal, rotating, solid disk). One child is 2 m from the axis of rotation and the other is 4 m from the axis. Which one of the following statements is correct? ω is the same, $v_T = r\omega$

(E)

(A) Both children have the same tangential velocity.
(B) The tangential velocity of the child nearer the axis is twice that of the child further from the axis.
(C) The angular velocity of the child nearer the axis is half that of the child further from the axis.
(D) The angular velocity of the child nearer the axis is twice that of the child further from the axis.
(E) The angular velocity is the same for both children.

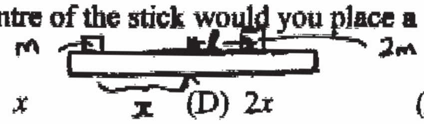
A7. You are standing on the edge of a spinning merry-go-round. The merry-go-round has a steadily decreasing angular velocity due to the presence of friction. Which one of the following statements regarding your total acceleration is correct? $\vec{a}_{tot} = \vec{a}_c + \vec{a}_T$ $\vec{a}_T$ is opposite to $\vec{v}_T$ b/c of slowing down.

(D)

(A) Your total acceleration is directed radially away from the center of the merry-go-round.
(B) Your total acceleration is directed radially toward the center of the merry-go-round.
(C) Your total acceleration is directed opposite to your tangential velocity.
(D) Your total acceleration has a component directed toward the center of the merry-go-round and a component directed opposite to your tangential velocity.
(E) Your total acceleration has a component directed toward the center of the merry-go-round and a component directed in the same direction as your tangential velocity.

A8. A uniform stick is balanced at its centre. If a mass m is now placed a distance x from the centre of the stick, at what distance from the centre of the stick would you place a mass of $2m$ in order to keep the stick balanced?

(B)



$\Sigma \tau = 0$

(A) ¼x (B) ½x (C) x (D) $2x$ (E) $4x$

$mgx - 2mgl = 0$

$l = x/2$

A9. Which one of the following statements concerning a rigid object in equilibrium is **NOT** correct?

(B)

(A) The sum of the externally applied torques must be zero. T
(B) The translational velocity must be zero. F
(C) The translational acceleration must be zero. T
(D) The net externally applied force must be zero. T
(E) The angular acceleration must be zero. T

A10. A spherical balloon has a moment of inertia of I_1 when inflated to a radius of r_1. If the balloon is inflated further, so that the radius doubles, what is the new moment of inertia, I_2, in terms of I_1? You may assume that the balloon is a hollow sphere and neglect the mass of the air in the balloon.

(D)

(A) $I_2 = I_1$ (B) $I_2 = 2I_1$ (C) $I_2 = 8\pi I_1$ (D) $I_2 = 4I_1$ (E) $I_2 = 4\pi^2 I_1$

$$I \propto r^2 \Rightarrow \frac{I_2}{I_1} = \frac{r_2^2}{r_1^2} = \frac{(2r_1)^2}{r_1^2} = 4$$

$$I_2 = 4I_1$$

continued on page 4...

PART B

FOR EACH OF THE FOLLOWING PROBLEMS, WORK OUT THE SOLUTION IN THE SPACE PROVIDED AND ENTER YOUR ANSWERS ON PAGE 6.

ONLY THE ANSWERS WILL BE MARKED. THE SOLUTIONS WILL NOT BE MARKED.

B1. A hawk moving at a speed of 9.50 m/s dives to an altitude 8.20 m lower. Calculate the hawk's speed at the lower altitude. You may ignore any effects due to air resistance and lift.

so $W_{nc} = 0$

$$E_o + W_{nc} = E_f$$

$$\tfrac{1}{2}\cancel{m}v_o^2 + \cancel{m}gh_o = \tfrac{1}{2}\cancel{m}v_f^2 + \cancel{m}gh_f$$

$$\tfrac{1}{2}v_o^2 + g(h_o - h_f) = \tfrac{1}{2}v_f^2$$

$$v_f = \left[v_o^2 + 2g(h_o - h_f)\right]^{1/2}$$

$$v_f = \left[(9.50\,\text{m/s})^2 + 2(9.80\,\text{m/s}^2)(8.20\,\text{m})\right]^{1/2} = 15.8\,\text{m/s} \quad \text{(D)}$$

B2. Calculate the average power output (in Watts) of a fly of mass 1.30×10^{-3} kg as it flies vertically upward at a constant velocity of 1.50 m/s.

$\therefore \ \Sigma\vec{F} = 0$ $\uparrow +$

$$\bar{P} = F\bar{v}$$

$$F_{fly} - W = 0$$

$$F_{fly} = W$$

$$\bar{P} = F_{fly}\bar{v}$$

$$\bar{P} = W\bar{v} = mg\bar{v} = (1.30 \times 10^{-3}\,\text{kg})(9.80\,\text{m/s}^2)(1.50\,\text{m/s})$$

$$\bar{P} = 1.91 \times 10^{-2}\,\text{W} \quad \text{(A)}$$

continued on page 5...

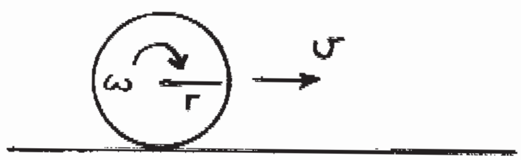
B3. A gymnast swings counter-clockwise on a high bar, making one complete revolution every 1.51 seconds. Calculate the average angular velocity of the gymnast in radians per second.

$T = 1.51s$ $\quad \bar{\omega} = \frac{\theta}{t} = \frac{2\pi \text{ rad}}{T}$

$$\bar{\omega} = \frac{2\pi \text{ rad}}{1.51s} = 4.16 \text{ rad/s}$$ (B)

B4. The tires of a car have a radius of 0.345 m and are rotating at an angular velocity of 79.4 rad/s. Calculate the speed of the car, assuming that the tires are rolling without slipping along the road surface.

$v = v_T = r\omega$



$v = (0.345 \text{ m})(79.4 \text{ rad/s})$

$v = 27.4 \text{ m/s}$ (E)

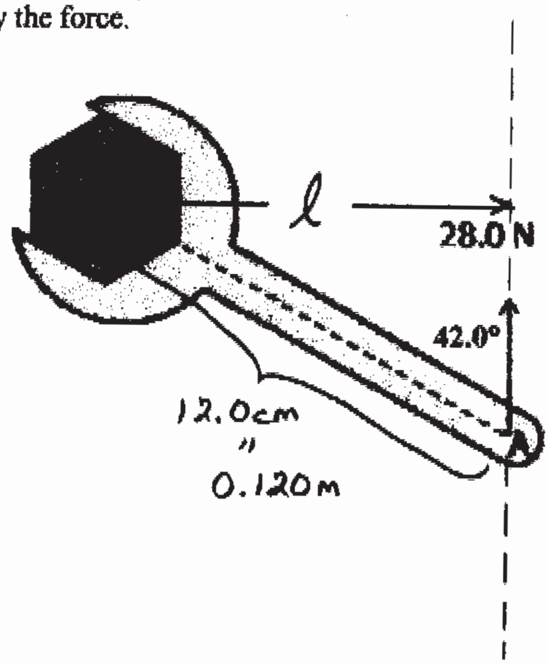
continued on page 6...

B5. A force of 28.0 N is exerted on the wrench at an angle of 42.0° relative to the line OA, as shown in the diagram. Given that point O is the axis of rotation, and that point A is a distance of 12.0 cm from O, calculate the torque exerted about point O by the force.

$\tau = F\ell$

$\tau = (28.0\,N)(0.120\,m \sin(42.0°))$

$\tau = 2.25\ N\cdot m$ (C)



ANSWERS FOR PART B

ENTER THE ANSWERS FOR THE PART B PROBLEMS IN THE BOXES BELOW.

THE ANSWERS MUST CONTAIN THREE SIGNIFICANT FIGURES AND THE UNITS MUST BE GIVEN.

ONLY THE ANSWERS WILL BE MARKED. THE SOLUTIONS WILL NOT BE MARKED.

B1	15.8 m/s	(D)
B2	1.91×10^{-2} W	(A)
B3	4.16 rad/s	(B)
B4	27.4 m/s	(E)
B5	2.25 N·m	(C)

continued on page 7...

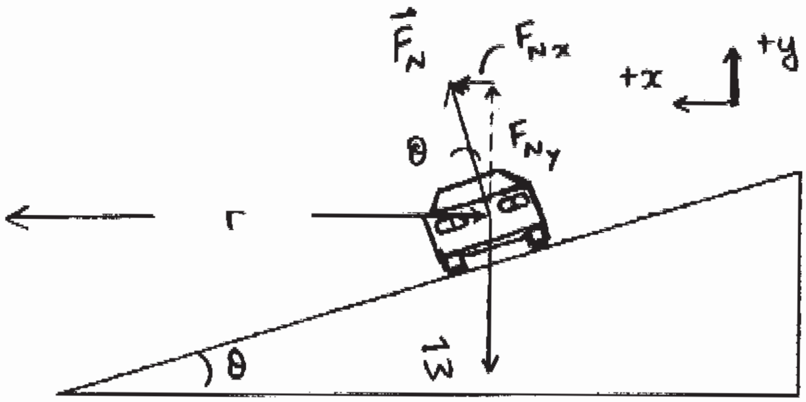
PART C

IN EACH OF THE FOLLOWING QUESTIONS, GIVE THE COMPLETE SOLUTION AND ENTER THE FINAL ANSWER IN THE BOX PROVIDED.

THE ANSWERS MUST CONTAIN THREE SIGNIFICANT FIGURES AND THE UNITS MUST BE GIVEN.

NO CREDIT WILL BE GIVEN FOR ANSWERS ONLY, <u>SHOW AND EXPLAIN YOUR WORK</u>. EQUATIONS NOT PROVIDED ON THE FORMULA SHEET MUST BE DERIVED.

C1. A car goes around a curve on a road that is banked at an angle of 30.0° with the horizontal. Assume that friction is negligible (the road is very icy), and that the car will just stay on the road and make it around the curve when its speed is 24.0 m/s. Calculate the radius of the curve.



end (cross-section) view of road

$\boxed{102\text{ m}}$

$\vec{a}$ is horizontal (a_c), so choose x-y axes horizontal and vertical. Then $\Sigma F_x = ma$; $\Sigma F_y = 0$

$\Sigma F_x = ma_c$

$F_{Nx} = \frac{mv^2}{r}$

$F_N \sin\theta = \frac{mv^2}{r}$

$\Sigma F_y = 0$

$F_{Ny} - W = 0$

$F_{Ny} = W$

$F_N \cos\theta = mg$

$F_N = \frac{mg}{\cos\theta}$

$\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{mv^2}{r}$

$\cancel{m}g\tan\theta = \frac{\cancel{m}v^2}{r} \Rightarrow r = \frac{v^2}{g\tan\theta}$

$r = \frac{(24.0\,m/s)^2}{(9.80\,m/s^2)(\tan 30.0°)} = 102\,m$

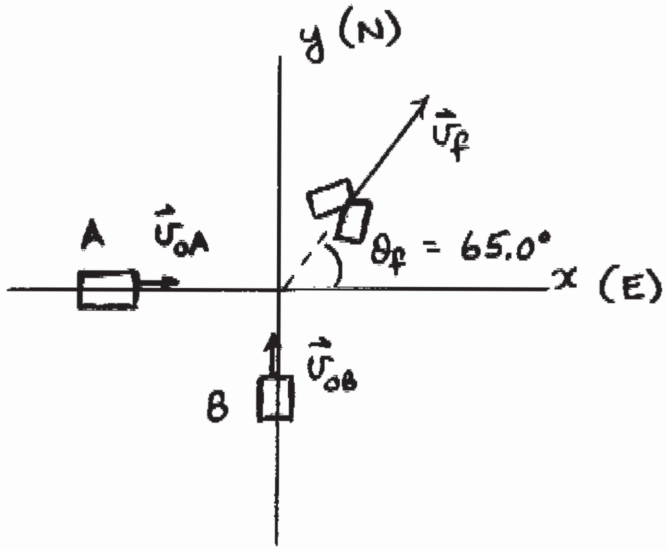
continued on page 8...

Stu. No.: ____________________

C2. Two cars collide at an intersection. Car A, of mass 2.16×10^3 kg, is initially travelling from west to east, while car B, of mass 1.59×10^3 kg, is initially travelling from south to north at 14.5 m/s. As a result of the collision the cars become locked together and move as one object after the collision. In your role as an expert witness you inspect the accident scene and determine that, after the collision, the cars moved at an angle of 65.0° north of east from the point of impact.

Calculate the speed of car A just prior to the collision.

$$\boxed{4.98\ \text{m/s}}$$



Ignore the effects of any external forces over the short time interval of the collision.
Linear momentum is conserved.

$$\vec{p}_{tot_f} = \vec{p}_{tot_o}$$

x-dir'n: (E)

$$p_{tot_{fx}} = p_{tot_{ox}}$$

$$(m_A + m_B) v_f \cos\theta_f = m_A v_{oA} + 0$$

$$v_{oA} = \frac{(m_A + m_B) v_f \cos\theta_f}{m_A}$$

y-dir'n (N)

$$p_{tot_{fy}} = p_{tot_{oy}}$$

$$(m_A + m_B) v_f \sin\theta_f = 0 + m_B v_{oB}$$

$$(m_A + m_B) v_f = \frac{m_B v_{oB}}{\sin\theta_f}$$

$$v_{oA} = \frac{m_B v_{oB} \cos\theta_f}{\sin\theta_f \, m_A} = \frac{(1.59 \times 10^3\ \text{kg})(14.5\ \text{m/s})(\cos 65.0°)}{(\sin 65.0°)(2.16 \times 10^3\ \text{kg})}$$

$$v_{oA} = 4.98\ \text{m/s}$$

continued on page 9...

C3. Suppose that the partial melting of the polar ice caps increases the moment of inertia of the earth from $0.331 M_E R^2$ to $0.332 M_E R^2$, where R is the radius of the earth and M_E is the mass of the earth.

Calculate, in seconds, the change in the length of the day. You may assume that there is no net external torque acting on the earth.

$\sum \tau_{ext} = 0$ so angular momentum is conserved.

$\boxed{261\,s}$

$I_f \omega_f = I_o \omega_o$

ω = angular velocity of rotation of Earth

$I_f \frac{\cancel{2\pi}}{T_f} = I_o \frac{\cancel{2\pi}}{T_o}$

$\omega_o = \frac{2\pi}{T_o}$ where $T_o = 24h$

$$\frac{0.332 \cancel{M_E R^2}}{T_f} = \frac{0.331 \cancel{M_E R^2}}{T_o}$$

$$T_f = \left(\frac{0.332}{0.331}\right) T_o$$

change in period, $T_f - T_o$, $= \left[\left(\frac{0.332}{0.331}\right) - 1\right] T_o$

$$T_f - T_o = \left[\left(\frac{0.332}{0.331}\right) - 1\right] \cdot 24h \cdot \frac{3600s}{h} = 261\,s$$

END OF EXAMINATION